Model Risk

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CORPORATE & INVESTMENT BANK

Contents

Models in Finance

- Models are used for many different purposes:
  - Valuation of structured products (Pricing)
  - Computation of capital requirements (VaR)
  - Design of risk measures (Stress scenarios)
  - Anticipation of client behavior (Loan prepayment)
  - Evaluation of the credit quality of a counterparty (Credit rating)
  - Estimation of future prices (Prediction)
  - Any suggestion on your side ?

Model Risk

- Models are all approximations of the real phenomenon
- All models rely on a set of hypothesis and even the simplest ones can be (or can become) flawed !
- Inevitably, wrong model outputs or inappropriate use of such outputs result in adverse consequences:
  - inaccurate valuation
  - mismanagement of complex portfolios
  - poor business decisions based on wrong risk measures
  - unanticipated liquidity shortfall
  - poor capital allocation
  - inadequate credit risk assessment
- Active Model Risk management is critical !

Model Risk

A simple example: Deposit in CHF

CHF Deposit Rates				
Today	09/03/1987	# days	Bid	Offer
O/N	10/03/1987	1	1,00%	1,125%
T/N	11/03/1987	1	1,00%	1,125%
1 W	18/03/1987	7	1,00%	1,125%
1 M	13/04/1987	33	3,00%	3,125%
2 M	11/05/1987	61	3,00%	3,125%
3 M	11/06/1987	92	3,0625%	3,1875%
6 M	11/09/1987	184	3,125%	3,25%
12 M	11/03/1988	366	3,25%	3,375%

Question: What would you quote a client who wants to lend 10M CHF from 11/03 to 25/05 ?

Model Risk

- Any suggestion ?

CHF Deposit Rates				
Today	09/03/1987	# days	Bid	Offer
O/N	10/03/1987		1,000%	1,125%
T/N	11/03/1987	0	1,000%	1,125%
1 W	18/03/1987	7	1,000%	1,125%
1 M	13/04/1987	33	3,000%	3,125%
2 M	11/05/1987	61	3,000%	3,125%
	25/05/1987	75		
3 M	11/06/1987	92	3,0625%	3,1875%
6 M	11/09/1987	184	3,125%	3,250%
12 M	11/03/1988	366	3,25%	3,375%

Model Risk

- First method that comes to mind: linear interpolation
- A very simple model !

3%+ 0,0625%*(75-61)/(92-61)= **3.03%**

Model Risk

- First method that comes to mind: linear interpolation
- A very simple model !

3%+ 0,0625%*(75-61)/(92-61)= 3.03%

- Reality is rather more complex ….

In Switzerland, bank regulatory reserves* used to be computed only at month end, so short term rates tend to be higher over the end of the month and significantly lower between two month ends!

This yields to a different calculation:

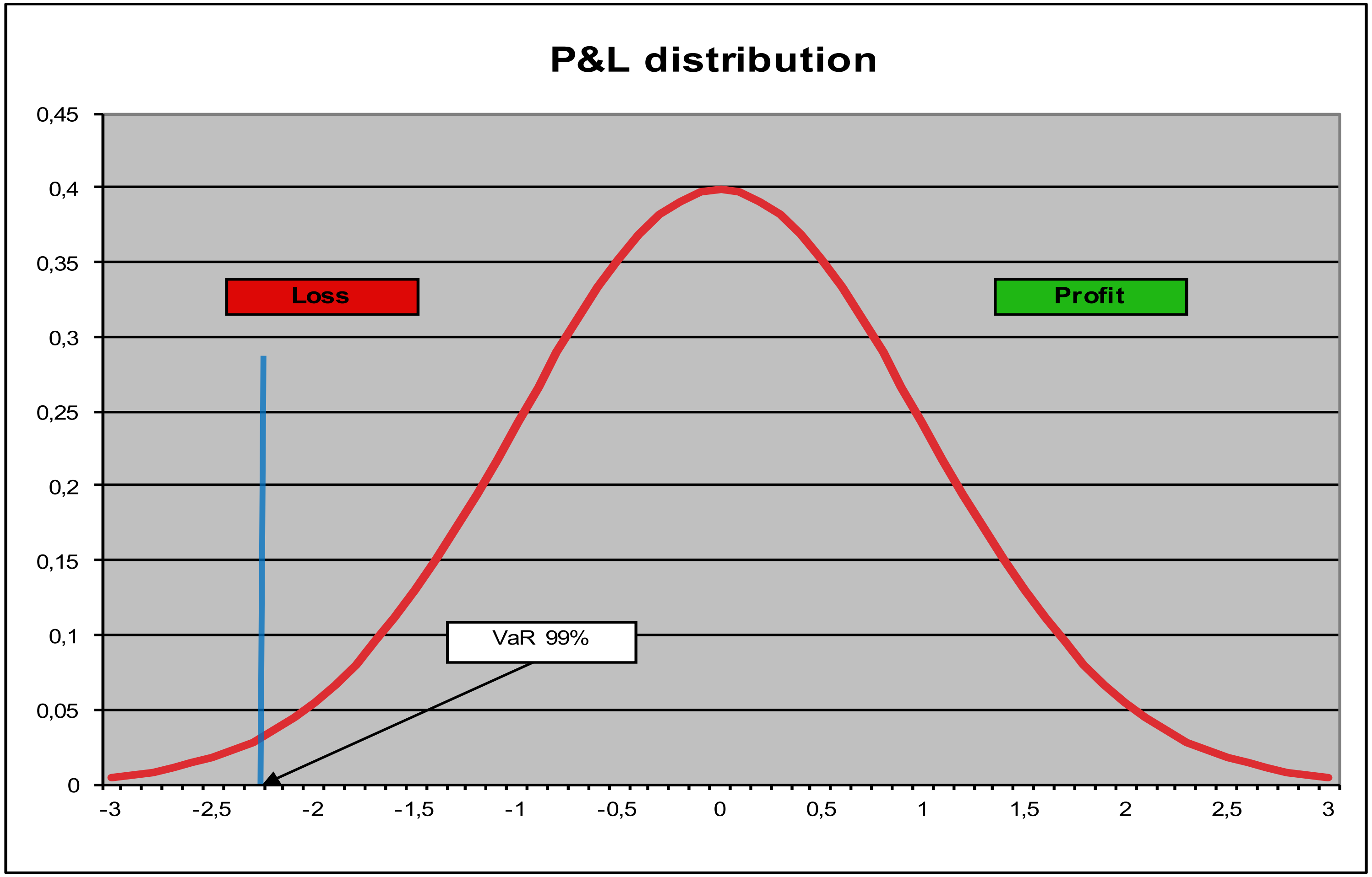
(61*3%+14*1%)/75= **2,63%**

Result: On a 10 M CHF deposit, you just lost 8 333 CHF!

* Regulatory Reserves is a deposit that a commercial bank needs to leave at the Central Bank defined as a proportion of the amount of loans made to customers

Value at Risk

- To measure Capital Requirement for Market Risk, major banks use an Internal Model based on VaR 99% 1-day



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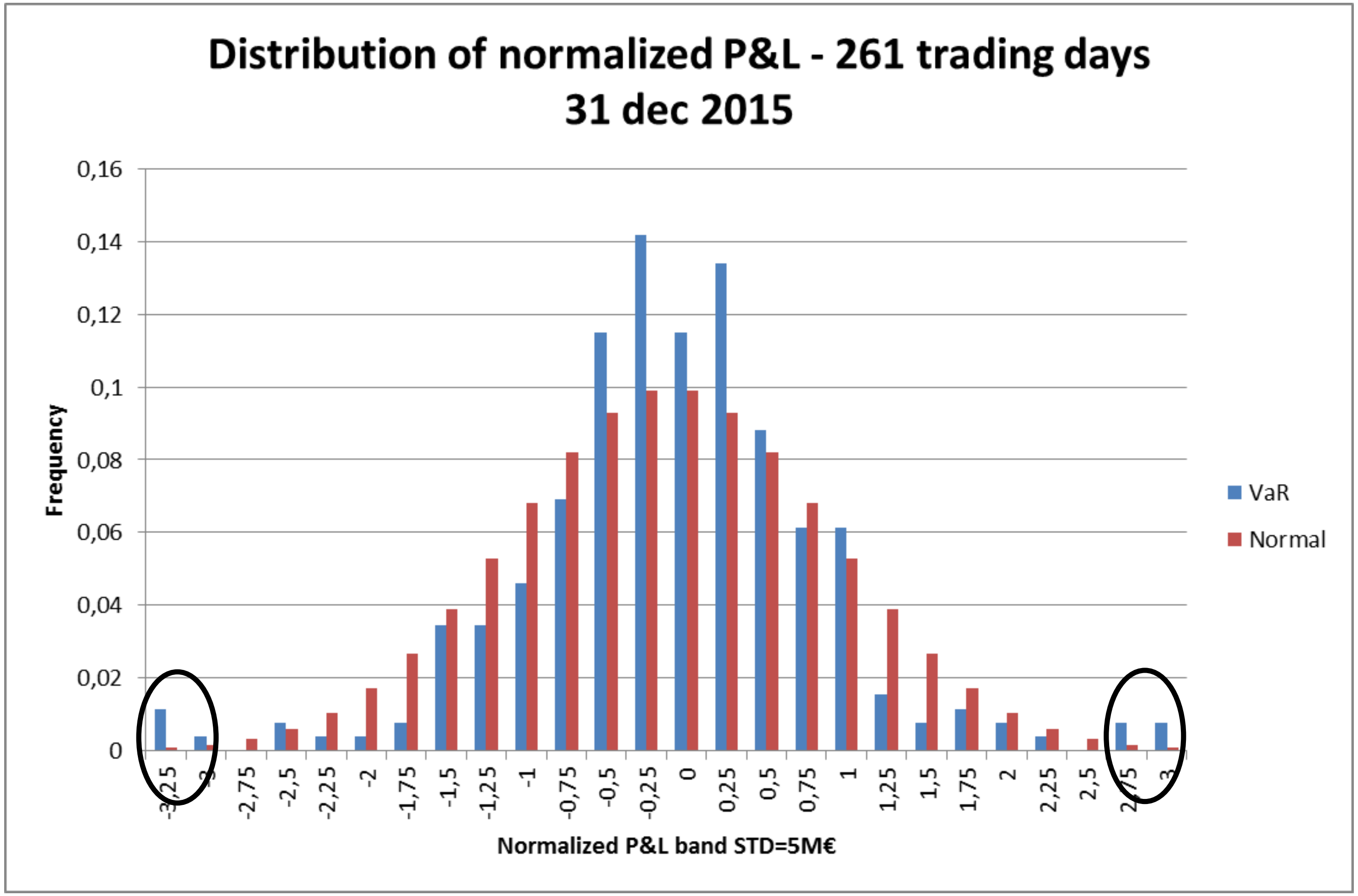
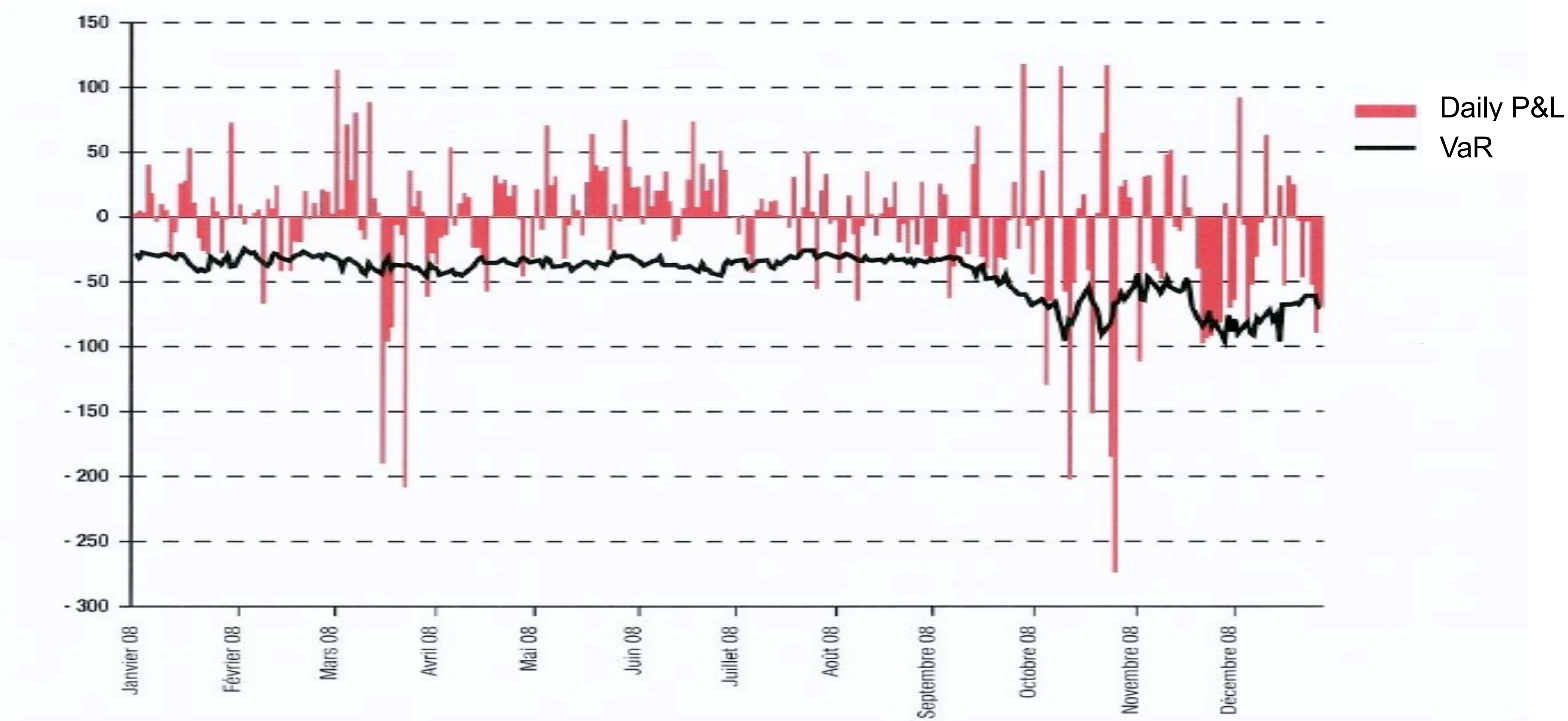
Value at Risk

Empirical Distribution versus Gaussian hypothesis



VaR during 2008 Crisis

VaR 99% 1d during the 2008 Crisis: 29 Back-testing exceptions !



New regulatory framework : Basel 2.5 (2012): Stressed VaR;

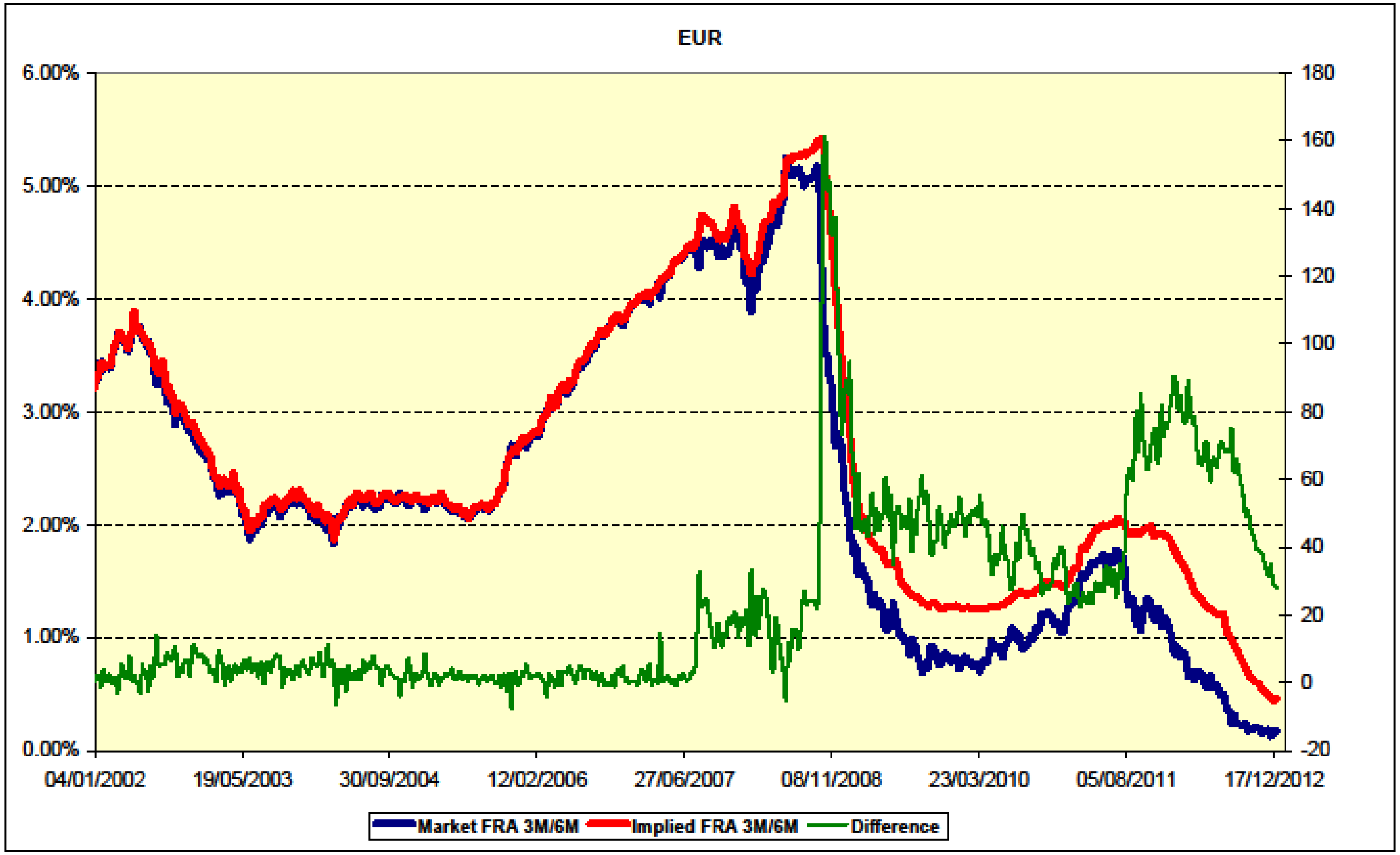
Basel 3 (2025): Fundamental Review of the Trading Book (FRTB)

Mono Curve Pricing Framework

- FRA 3/6 EUR Theoretically in a mono-curve framework:

$$(1+n_{FRA}/360*FRA3/6)=(1+n_{6m}/360*Euribor6)/(1+n_{3m}/360*Euribor3)$$

n_{FRA}: number of days between start date and end date of the FRA (Forward Rate Agreement)



➡ New Pricing Framework: Multi Curve

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Bond Pricing Model

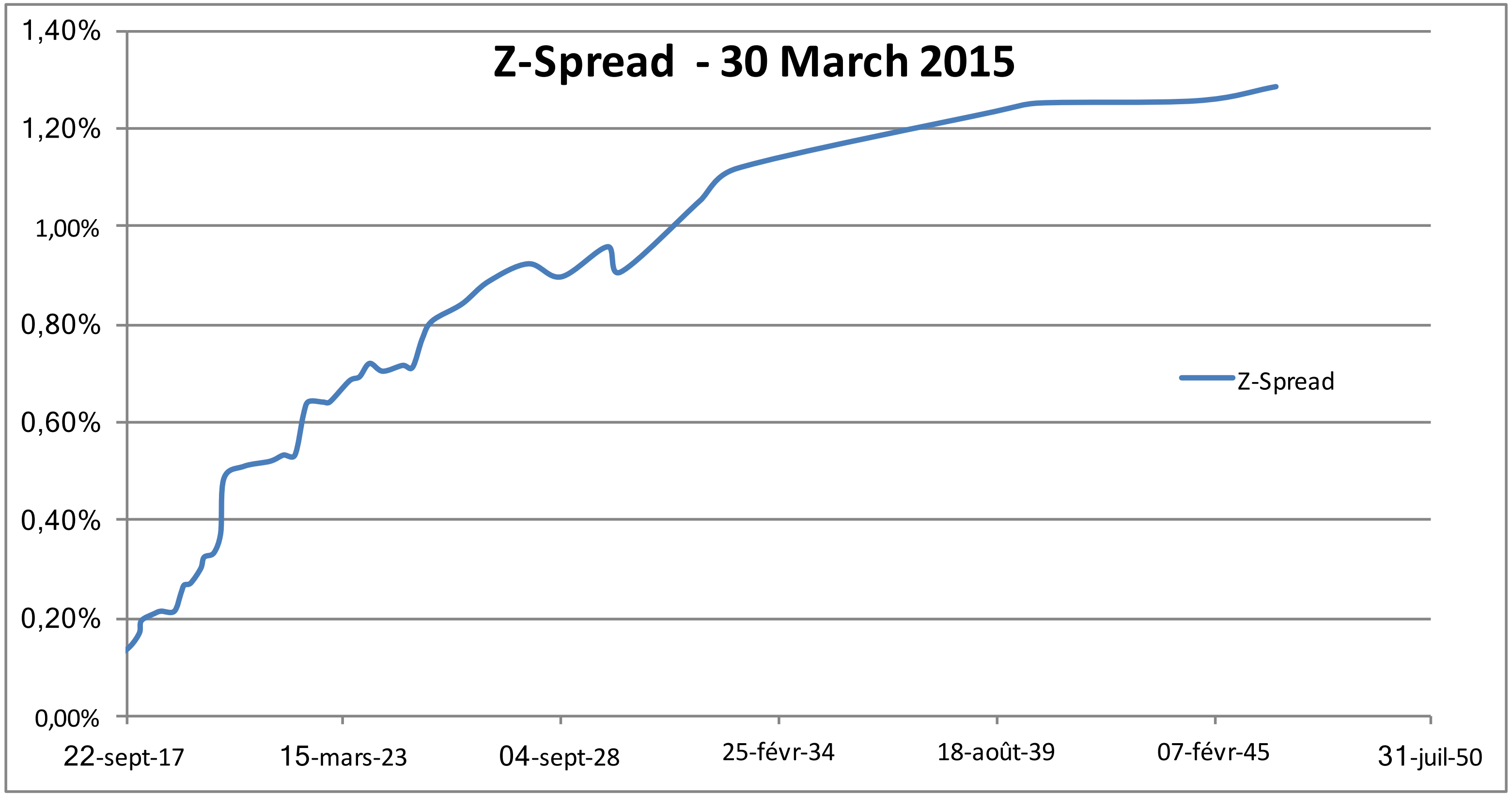
- Bond pricing: sounds easy !
  - discount all future cash flows with a (credit) spread for a given issuer and maturity
  - a popular approach is to use a z-spread, equivalent to credit spread over zero-coupon risk free rates

$$B(t_0, T) = \sum_{i=1}^{N} C_i \times D(t_0, t_i) e^{-z_B(t_i - t_0)} + D(t_0, t_N) e^{-z_B(t_N - t_0)}$$

C_i: fixed or floating coupon for date t_i
z_B : z−spread for bond B
$D(t_0, t_i)$: discount factor of the risk free reference curve for a cash flow paid at t_i

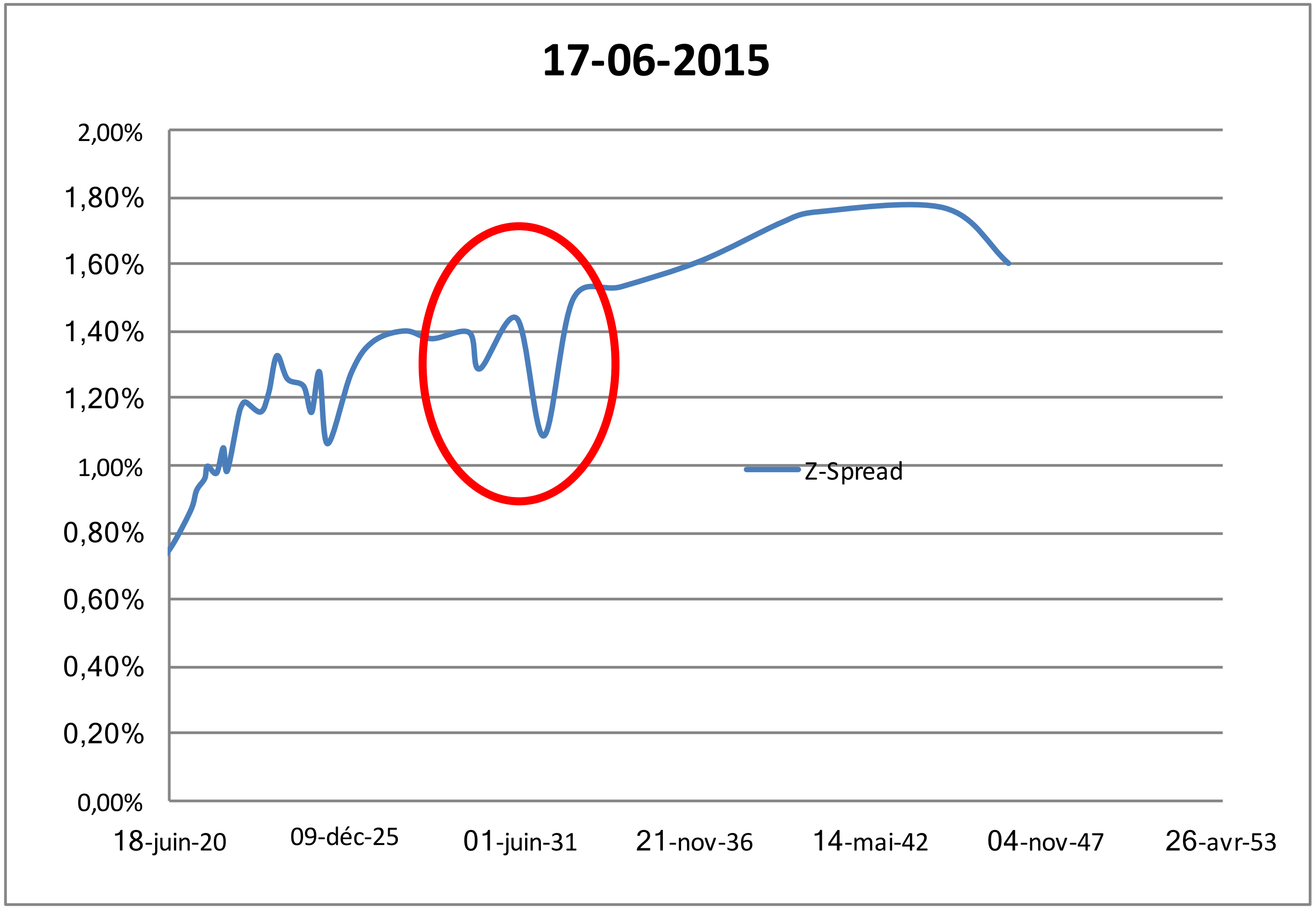
Bond Pricing Model

- Italian Government bonds z-spreads over Euribor 6-month for bonds covering a large spectrum of maturities



Bond Pricing Model

- Let's move forward a few months !



→ Inconsistent behavior of z-spread for 2030 – 2035 maturities!
Question: alternative valuation approach ?

Bond Pricing Model

Alternative pricing model

- an alternative valuation process is proposed, inspired by a credit framework: default probabilities and a recovery rate are integrated in the valuation process

$$\bar{B}(t_0, T) = \sum_{i=1}^{N} C_i D(t_0, t_i) Q(t_0, t_i) + D(t_0, t_N) Q(t_0, t_N) + RR \int_{t_0}^{t_N} D(t_0, u) dP(t_0, u)$$

$\mathrm{Q}(\mathrm{t}_0, \mathrm{t}_\mathrm{i})$: survival probability of issuer at time t_i
$\mathrm{P}(\mathrm{t}_0, \mathrm{u})$: default probability of issuer at time u
RR: Expected Recovery Rate as a % of the principal

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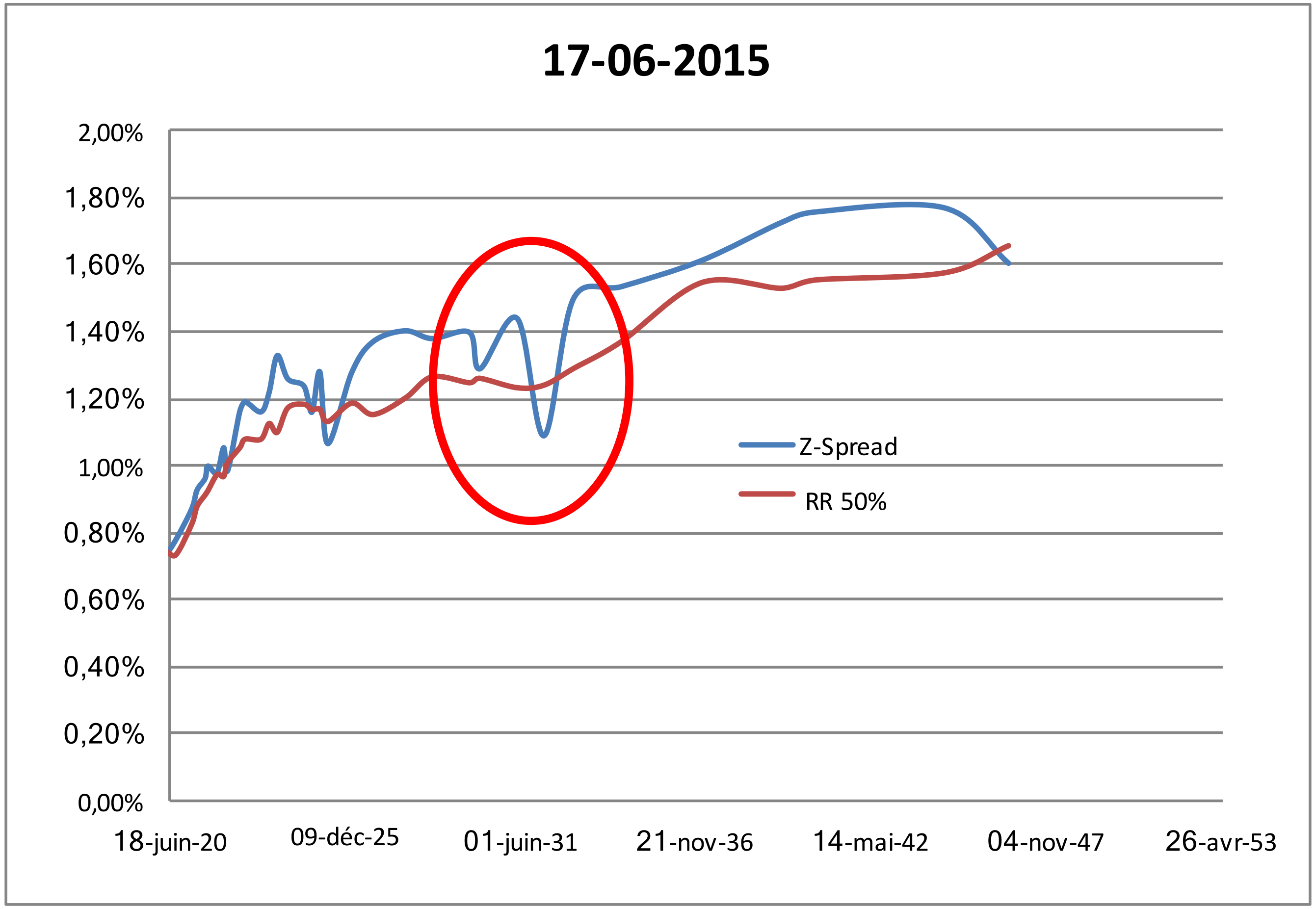
Bond Pricing Model

- Let's look again at bonds values by maturity !



→ In this new framework, bonds spread hierarchy is coherent!

Valuation Model

Valuation of Structured Products

- When valuing structured products, a model is basically:
  - a mathematical approximation of the joint dynamic of the risk factors to which the value of the product is dependent
  - calibrated to liquid market instruments for which we have tradable prices (bid and offer)
  - used to define a dynamic replication portfolio made of those liquid calibration instruments that will mimic locally the dynamic of the structured product. The objective of designing such a hedge portfolio is to minimize the impact of risk factor moves on the aggregated value of the hedge portfolio and the structured product
- The valuation of structured products is a relative concept
- As for all models, valuation models generate model risk

Risk Neutral Probability

- The concept of risk neutral probability is often used to simplify the valuation. What does it mean ? How does it work ?
  - If a risk factor (spot price) can be hedged continuously with tradable instruments in complete markets and these instruments are correctly priced in the model (calibration), then:
  - as a result of the absence of arbitrage opportunity (AAO), the portfolio made of the structured product and the hedge yields the risk free rate.
  - The valuation can directly use the risk neutral probability (without any market price of risk) to value the structured product.

Black & Scholes Model

- Example 1 : B&S model, equity option. By hypothesis the volatility is supposed to be constant (over time, strikes and maturities)

$$dS_t = S_t(\mu dt + \sigma dW_t)$$

  - In this equation μ, the drift, is unknown. Since one can use the forward equity price to delta hedge, μ can be replaced by the market expected drift (repo-div).

$$dS_t = S_t\{(repo - div)dt + \sigma dW_t\}$$

  - However, the volatility σ is unknown and must be estimated. It will be a source of risk because the constant volatility hypothesis is obviously flawed !

Stochastic Local Volatility Model

- Example 2: Stochastic local volatility model , FX options

$$dS_t = \mu S_t dt + S_t V(t, S_t)\sigma_t dW_t^{(1)}$$

$$d\sigma_t = \alpha_t \sigma_t dW_t^{(2)}$$

$$\left\langle dW_t^{(1)}, dW_t^{(2)} \right\rangle = \rho dt$$

- If there is a complete market for vanilla options, the volatility is no longer an estimation but becomes a tradable parameter and it can be hedged by buying/selling the most adapted liquid options.
- Correlation (ρ) and volatility of volatility (α) remain unknown and must be estimated and will remain sources of risk even if their impact is less critical for most structured products.

Model Risk in the context of Valuation

- Model risk can be:
  - a range of prices achievable under various acceptable models (R. Cont)
  - “the risk that the model price is or will be in the future significantly different from the market price, when such price is revealed” (R. Rebonato)
  - the risk that the “standard” market model jumps from one model to another (usually) more complex model (multi-curve pricing)

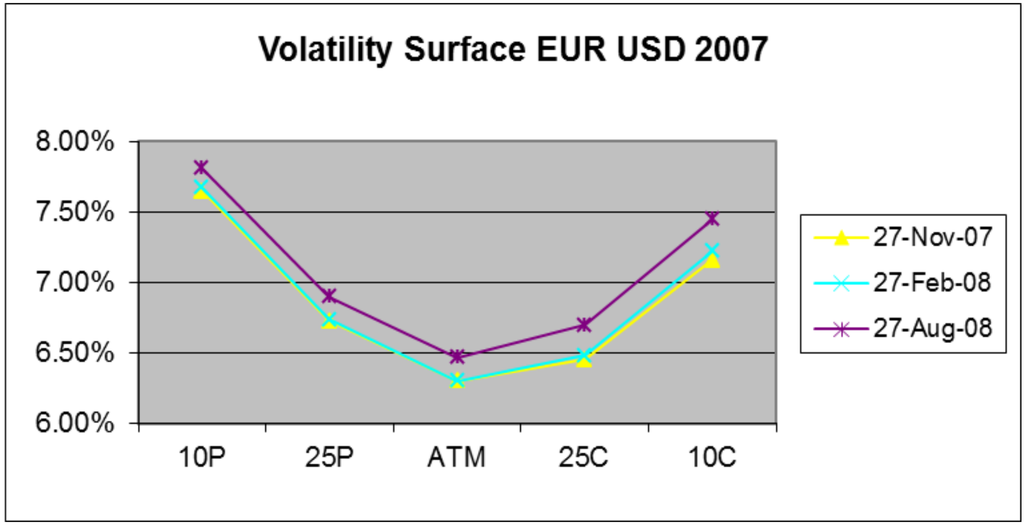
Model Risk Mitigation

- Accounting Reserves and Capital Buffers (Prudent Valuation)
  - Can be defined by introducing
    - alternative calibration sets
    - alternative numerical methods
    - confidence interval for estimated parameters
    - alternative models
  - Quantitative responses
    - M Avellaneda (1995): worst case scenarios on volatilities
    - R Cont (2006): worst case scenarios on family of models
    - P Henaff (2010): conditional worst case scenarios

Model Risk – Alternative Models

Stochastic Local Volatility model for FX exotic options

- Prices for a range of values of $\rho \in [-50\%, 50\%]$ and $\alpha \in [0\%, 150\%]$



EUR/USD 2007	B&S	Dvega	LV	Max	Min	Model Risk
Double No Touch	71,89	69,02	67,23	69,96	67,16	**4,1%**
One Touch	84,22	81,47	80,75	81,94	80,74	**1,5%**
Digital European	89,73	88,36	88,60	88,60	88,49	**0,1%**
Fwd Call	0,69	0,89	0,88	0,91	0,78	**15,0%**
Down & Out Call	0,58	0,73	0,73	0,74	0,70	**5,5%**
Down & Out Hurdle Call	0,58	0,76	0,74	0,76	0,72	**5,3%**
Range	13,45	7,53	8,72	8,74	7,99	**9,0%**
Lookback	6,2	3,6	4,0	4,0	3,7	**8,9%**

Model Risk

(Max-min)/(Max+min)*2

Usual Model Shortcomings !

- One way market
  - There is always a buyer and a seller but…
  - …one has a static view (investor), the other has a dynamic view (structured desk)
- Mark-to-Market and Mark-to-Model consistency
  - complex products become tradable (Convertible Bonds, CMS swaps)
  - market information can be collected about unobservable model parameters (consensus) or model output prices (collateral valuations)
- Missing risk factor in the model
- Unrealistic model assumptions
- Concentration of exposures and market impact

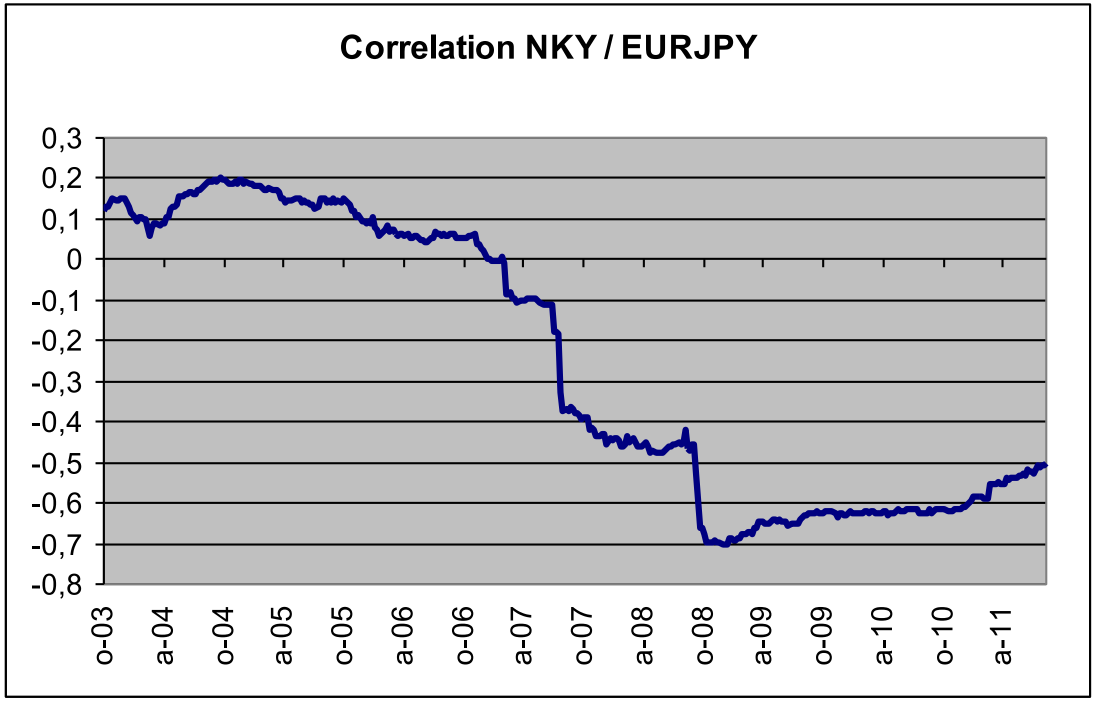
Valuation Model Issues

- Parameter estimation
- Missing Risk Factor
- Mark-to-Market and Mark-to-Model discrepancies
- Change of paradigm
- Gamma feedback

Parameter Estimation

- Quanto Equity Call : Max $(NKY-K)^+$ paid in EUR
  - 2 new risk factors :
    - Correlation between Nikkei and EURJPY
    - Volatility EURJPY (observable)

- Estimation procedure
  - historical : -50%
  - worst case : -70%
  - Market : ?



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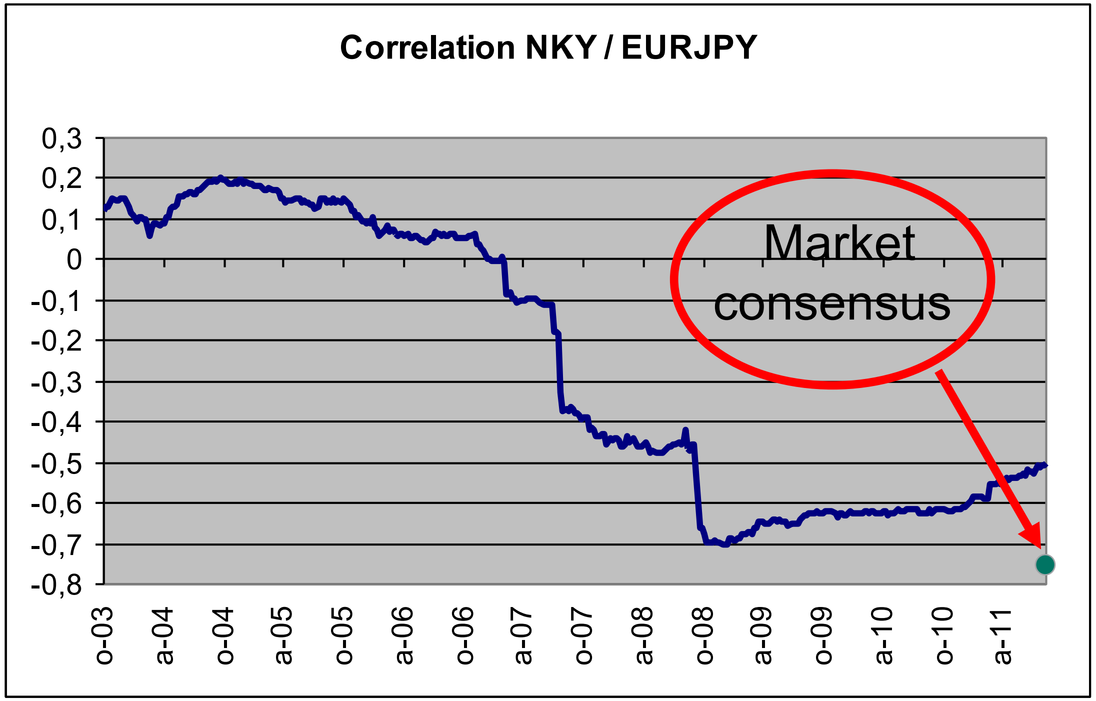
One way market !

The “market” price of the correlation is very conservative



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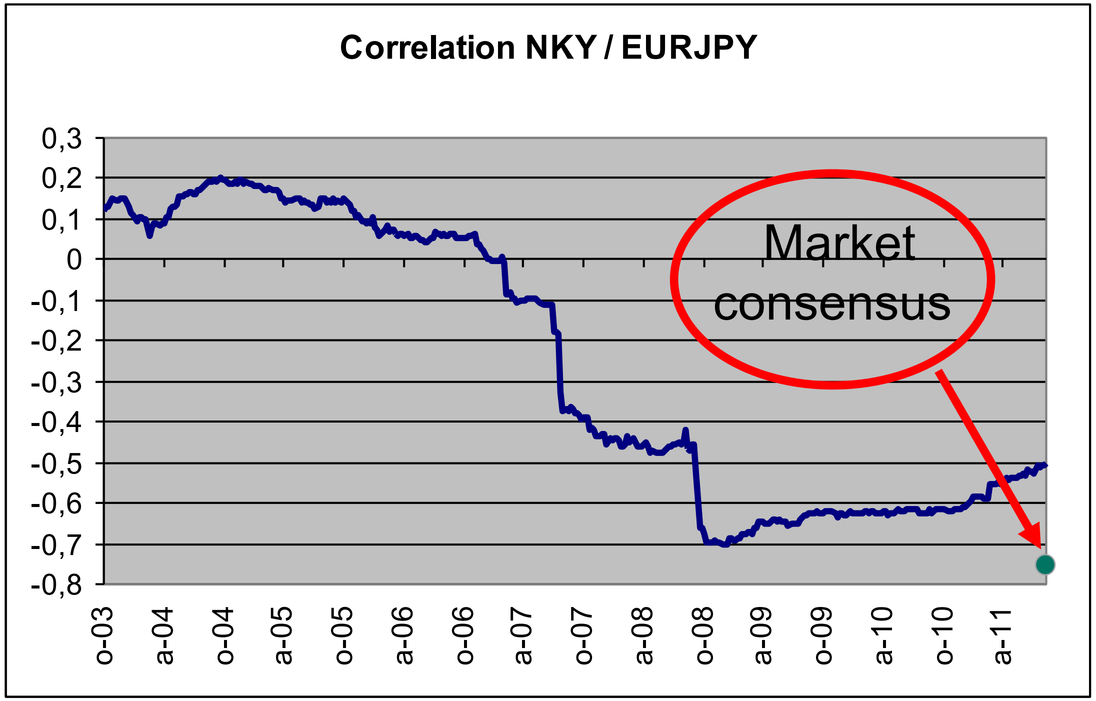
Question: Any comment on the correlation ?



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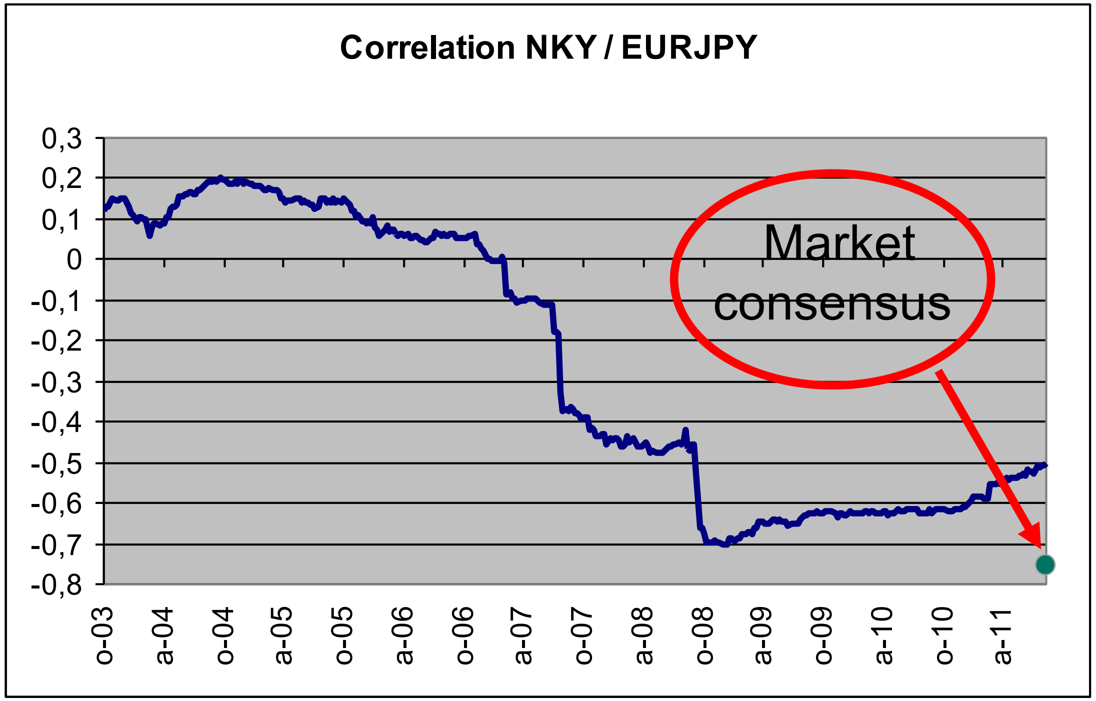
Question: What is the model hypothesis that is not verified ?



Credit Derivative

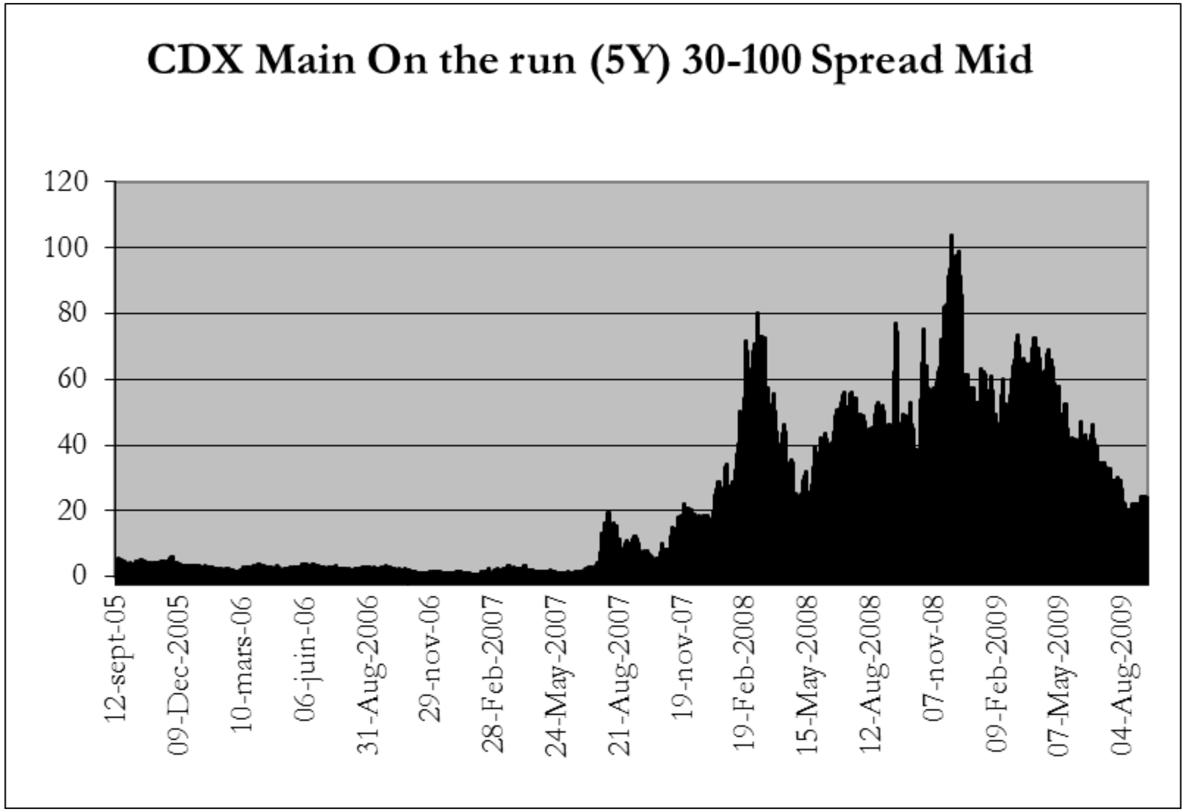
- a CDO tranche is characterized by:
  - A basket of CDS (typically 100 to 125 names)
  - An attachment point A
  - A detachment point D
  - A maturity date until which default are registered
  - It is represented by the interval [A, D]

- The payoff is equal to the sum of the losses (L_i) on the basket of CDS minus the attachment point and capped at (detachment –attachment point)

$$L = \min\left\{ \max\left(\sum_{i=1}^{n} L_i - A; 0\right); D - A \right\}$$

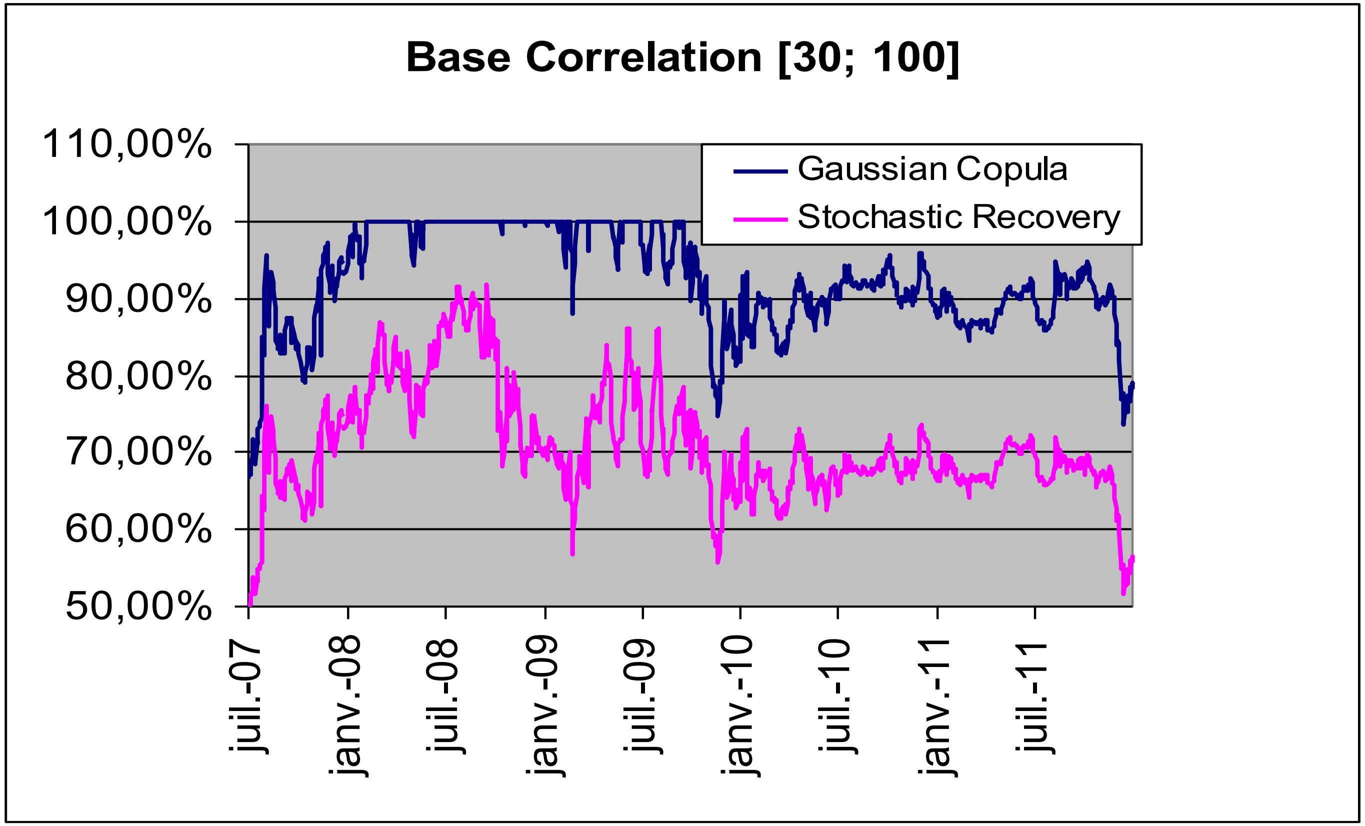
Credit Derivative

- Evolution of the market spread for tranche [30; 100]



Model risk: Missing Risk Factor

- Pricing Model : Gaussian Copula with a correlation for each tranche
  - Implied Base correlation for [30; 100]



➡ Market Price unreachable!

➡ Missing Risk Factor: Stochastic Recovery

Mark-to-Model vs Mark-to-Market

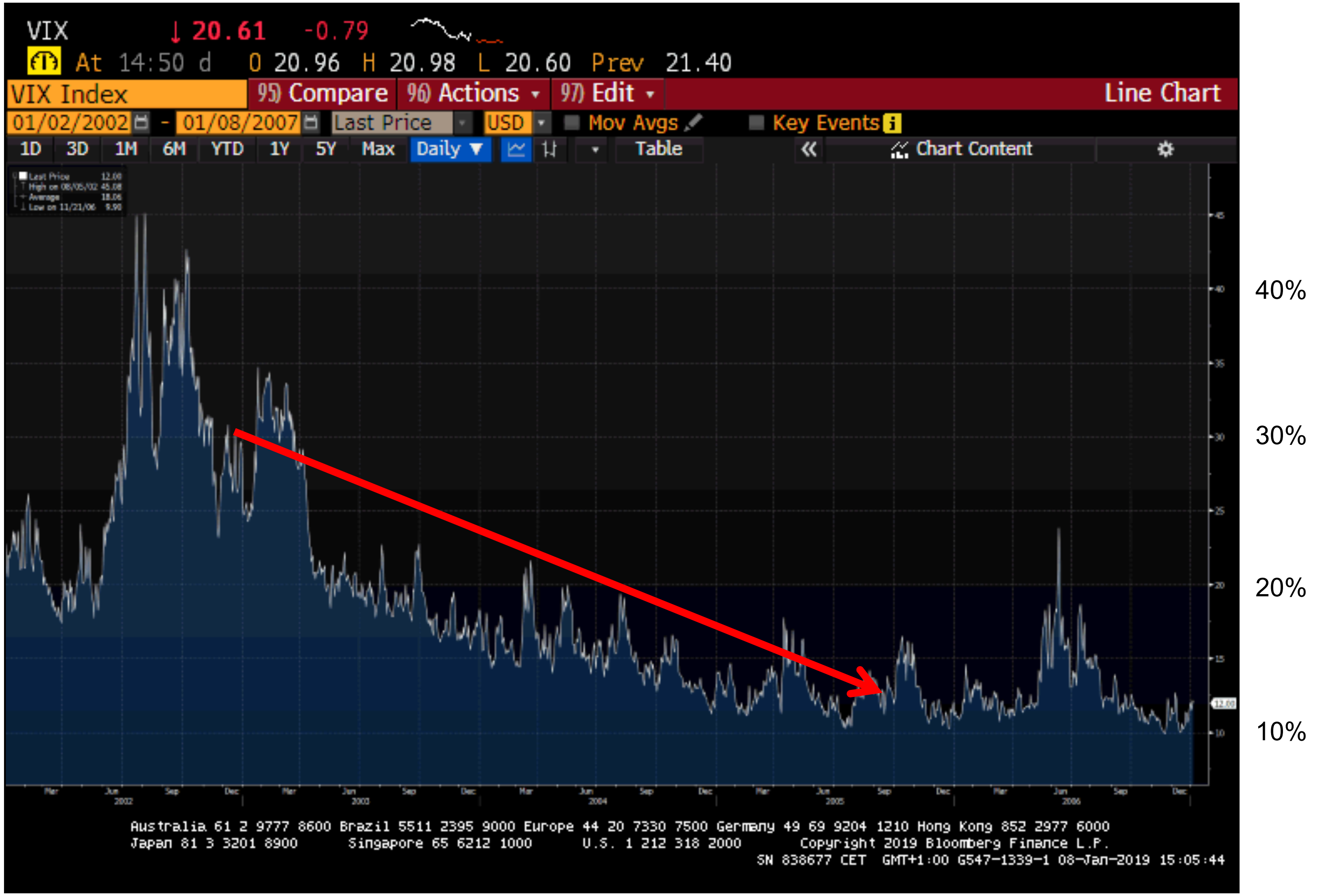
- Like all products, structured products need to be valued daily
  - hence on a daily basis they are mark-to-model (valued with a model) in the absence of an organized secondary market
  - if the product becomes tradable, then the bank must switch to a mark-to-market (remember Rebonato’s Model Risk definition)
  - some participants look at the evolution of prices between today and tomorrow (market makers); other look at a replication strategy until maturity (structured desks): valuations can be different for quite long periods !
  - even if there is a robust strategy to replicate the model price, because of risk limits (VaR, Stop loss), because of counterparty defaults, there will be situations where the bank will need to :
    - Unwind the position at the market price
    - Take the loss if the market price is lower than the model price

Mark-to-Model vs Mark-to-Market

- Convertible bond
  - Liquid market: market makers offer two-way prices mainly to asset managers specialized in convertible bonds funds
  - Prices can be obtained with very complex models covering multiple risk factors
    - Equity: price, volatility, dividend and repo rate
    - Credit spread of issuer
    - Interest rates
    - Correlations between all these risk factors
  - Model prices and market prices can diverge!
  - If there is a significant discrepancy watch out !

Mark-to-Model vs Mark-to-Market

- VIX - Volatility of S&P 500 (2002 – 2006)



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Mark-to-Model vs Mark-to-Market

- Mid 2002, the volatility starts to drop steadily affecting the value of all convertible bonds (bond + call)

- Model value falls below market value because fund managers buy from market makers to prevent prices to drop too much! *(to defend their position!)*

JACI - Japan Convertible Index

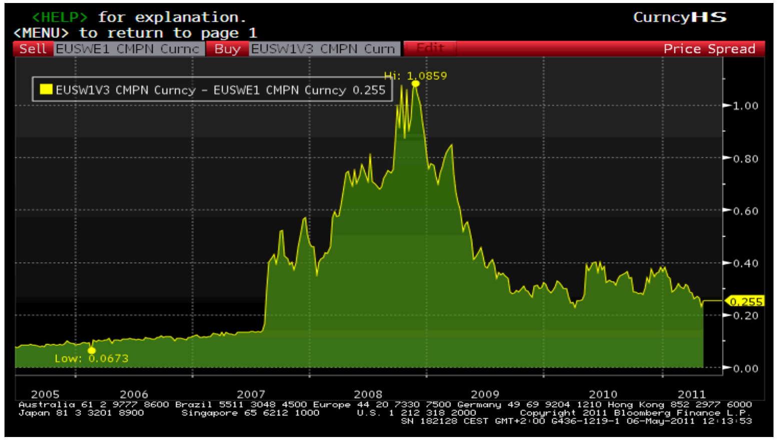
- It is difficult to sell short a convertible bond so the discrepancy can last …..

- At some point, market price will adjust!

Change of Paradigm

Collateralized derivatives

- The rate that is paid on collateral (OIS) is not the same as the one used to discount uncollateralized future cash flows (LIBOR)



Euribor 3m Swap 1y – OIS Swap 1y (2005-2011)

- Taking this difference between OIS and LIBOR into account changes significantly the value of long term collateralized derivatives (Piterbarg 2010). *C(u)* collateral received (posted when negative) at date *u.*

$$V(t) = E\left[e^{-\int_t^T r_{BOR}(u)du} V_T \right] + E\left[\int_t^T e^{-\int_t^u r_{BOR}(v)dv} \left(r_{BOR}(u) - r_{OIS}(u) \right) C(u)du \right]$$

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Change of Paradigm

Multi-Curve Pricing

- Pricing of interest rate products used to be derived from a single curve for products of all tenors.
- As liquidity spread between 3-month and 6-month tenor appeared during the 2008 crisis, this approach needed a complete rethink.

- In the new paradigm, each tenor requires its own forecast curve so the pricing requires:
  - a discount curve: OIS (collateral)
  - a 3-month forecast curve: LIBOR 3m
  - a 6-month forecast curve: LIBOR 6m
  - and so on ….

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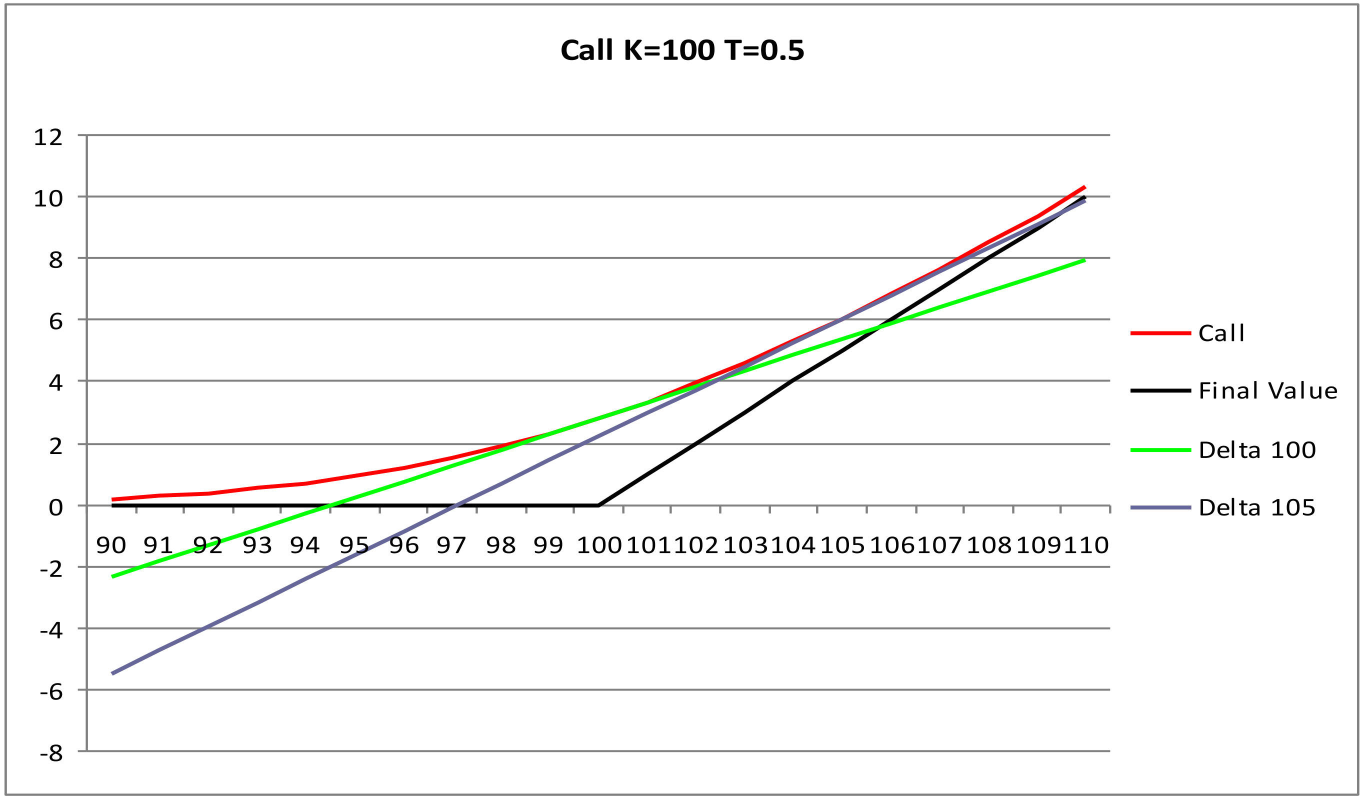
Gamma Feedback

- Hedging large derivative portfolios may impact market prices of hedging instruments (market impact)
- This affects principally illiquid risk factors: out-the-money volatilities, dividends, correlations …
- However, in some cases, even the price of the underlying risk factor can be affected
- When many structured desks are exposed to the same risk (one way markets) the potential for a jump of the price is magnified
  - the price impact will change the hedging requirement of all participants and that will put even more pressure on the price of hedge instruments (negative gamma)
  - this generates a feedback loop that can be devastating !

Gamma Feedback

- Theoretical example
  - Bank sold a call option on a single stock for a large size
  - negative gamma exposure:

→ when the stock price rises, the trader buys stocks to hedge

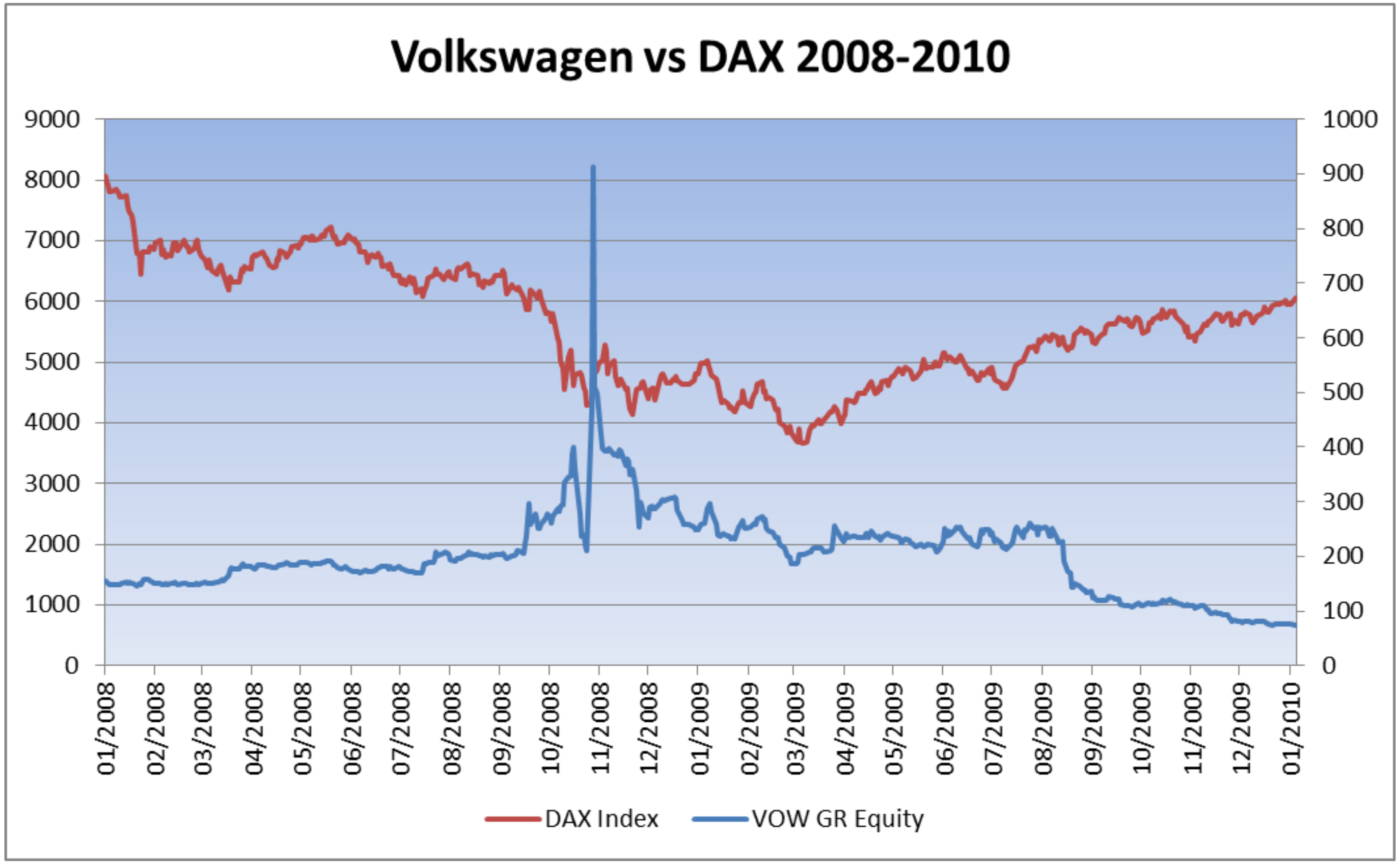


Gamma Feedback

- Theoretical example
  - If the amount of stock to buy is large, the market impact will be significant and the stock price will increase
  - a higher stock price changes the hedge requirement
  - … and sparks more buying!
  - and then you discover that you are not the only bank short of the same option !

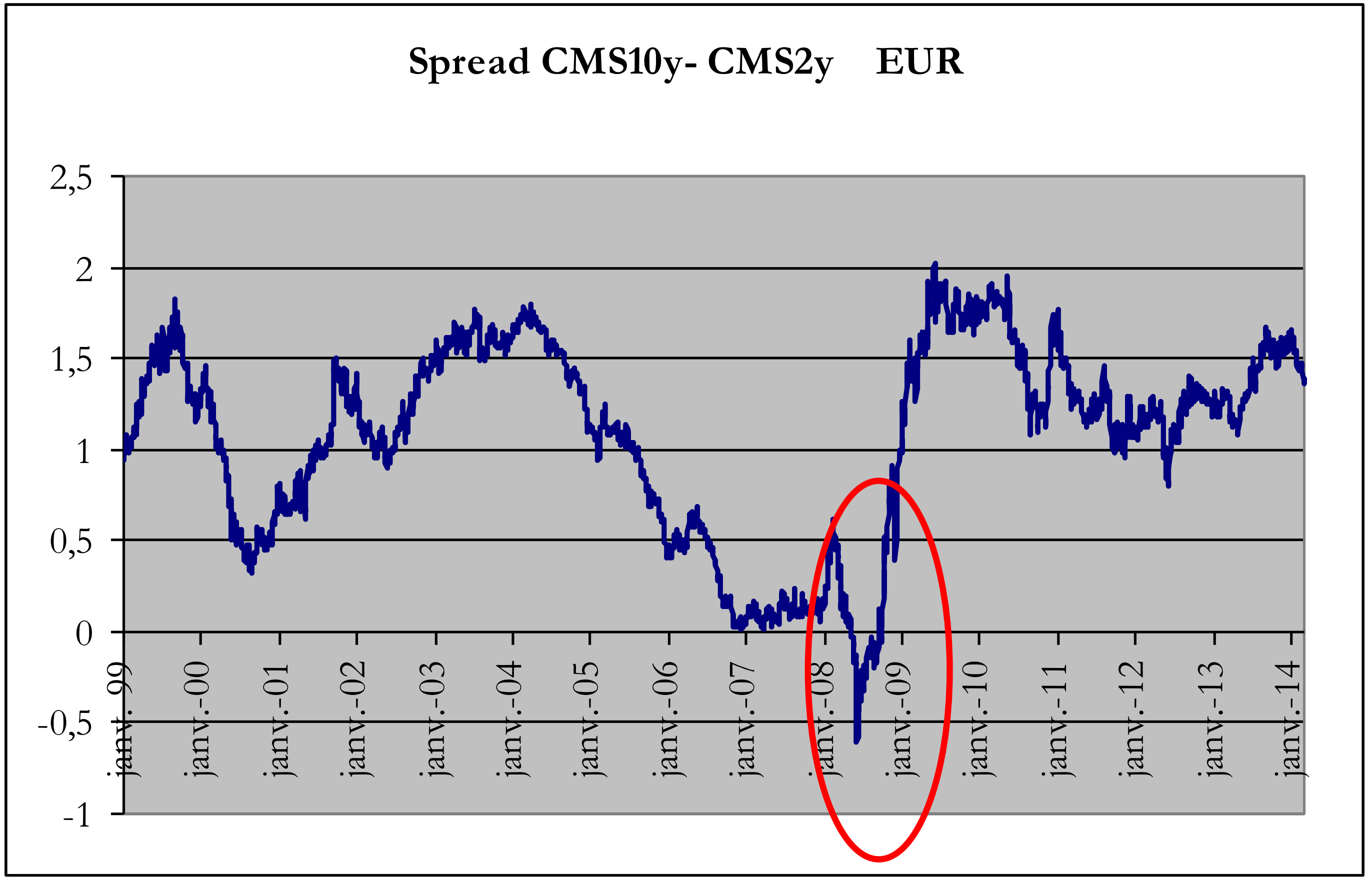
Gamma Feedback

- Not so theoretical after all !



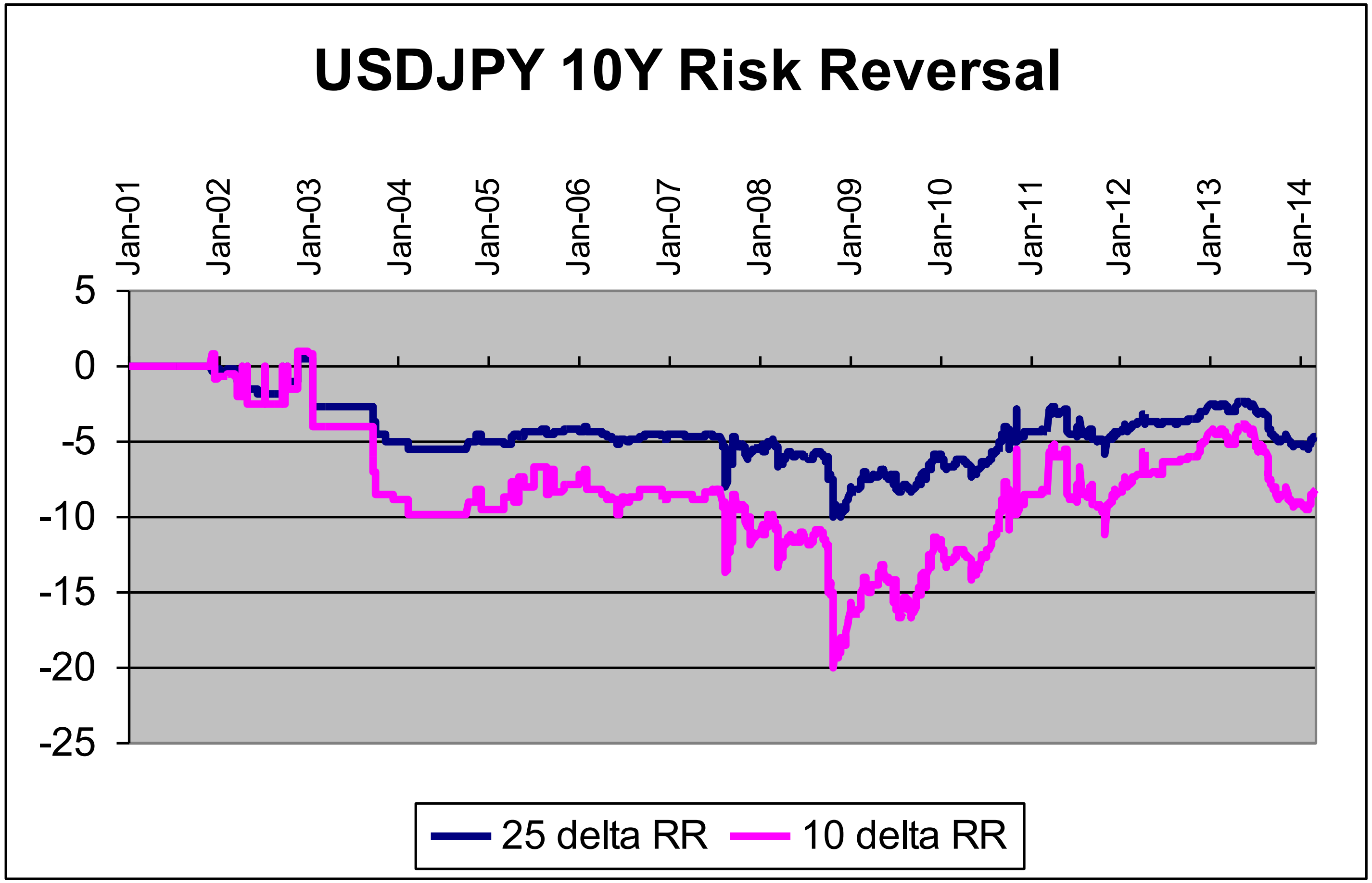
Gamma Feedback

- CMS spread derivative portfolios



Gamma Feedback

- Large USD/JPY long term derivative portfolio (PRDC)



Gamma Feedback: What can be done ?

- Difficult to detect because it will not appear in historical data until it is too late!
- The Front Office will argue that there is nothing to worry about!
- When the same structured product is sold to many clients, strict limits are required for un-hedgeable exposures
  - at some point the structured desk will have to hedge in some way otherwise they simply stop offering the product to clients
  - the bank is likely to be one of the first to hedge
  - hedging costs are not a linear function of size, so hedging a smaller position is always a lot cheaper
  - and when everyone else will rush to hedge, generating a large price impact, your losses have a better chance of remaining manageable

Model Risk Management Framework

- A Model Risk Management Framework should cover:
  - Model design
  - Model validation and periodic review
  - Governance, Policies and Procedures
  - Communication of model performance

Reference

OCC: Supervisory guidance on model risk management, 2011

Model Risk Management Framework

Design	Validation	Governance	Communication
• Purpose • Input Data • Mathematical motivation • Implementation • User validation • **Documentation**	• **Independence** • Competence • Authority • Relevance of Input Data • Mathematical soundness • Pertinence of model hypothesis • Implementation • Calibration • Robustness and stability • Model Parameter estimation • Validity Domain • Model Reserves	• Policies • Procedures • Allocation of resources • Annual Reviews • Model Inventory • Model Validation Committee • **Internal Audit**	• To **Senior Management and Board** • Model Performance • Backtesting • Benchmarking • Uncertainty linked to Model • Model Evolution

Key Takeaways

- Banks have made considerable investments in model risk management but models used by banks still exhibit significant shortcomings and hence risks
- Accounting reserves and capital buffers mitigate model risk to some extent
- Preventing accumulation of large exposures on hard-to-hedge risks even if they seem remote, is the only effective mitigation
- Always remember that models come with specific hypothesis that need to be continuously challenged and that critical judgment and regular reviews are as important as sophisticated mathematical representations of the financial markets.

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Model Design

- Statement of purpose
- Accuracy of input data
- Mathematical motivation
- Implementation, Calibration, Robustness
- Limitations
- User validation
- Documentation

Model Validation

- Three key features driving the model validation team's effectiveness:
  - Independence
    - Independent of model designer and user
    - Key to critical review
  - Competence
    - Skill, knowledge and expertise
    - Appropriate compensation and performance evaluation
  - Influence
    - Capacity to effectively challenge model design
    - Authority to require enhancements in a timely manner

Model Validation

- Evaluation of conceptual soundness
  - Mathematical soundness and accuracy
  - Relevance of the choice of input data (risk factors)
  - Evaluation of model hypothesis
  - Implementation, convergence of numerical resolution
  - Calibration to input data
  - Robustness and stability
  - Model parameter estimation
- Model risk mitigation
  - Define model reserves or/and capital buffers
  - Suggest more conservative features
  - Restrict model validity domain

Model Risk Mitigation

- Model reserves
  - Most banks use model reserves to mitigate model risk when valuing complex portfolios
  - However this will not cover dramatic changes in models (as changes of paradigm!)
- Capital requirement
  - For most risks (market, credit, operational) banks set capital aside according to regulatory rules or internal models.
  - Internal models are reviewed by the regulator who can require additional capital buffers if models are not adequate.
  - CRD IV: additional valuation adjustment for model risk may be required as a capital buffer on top of model reserves.

Model Validation

- Ongoing monitoring
  - Perform most of validation tests as an ongoing concern
- Output Monitoring
  - Sensitivity to model parameters
  - Benchmarking: compare model output with alternative model output or observable output
  - Back testing: compare model output and actual outcome

Model Validation

- Model Periodic Review
  - Market conditions change
  - Model hypothesis become unrealistic
  - “Standard” market models change

➡ All this imposes a model periodic review

- Annual review process that determines which model will be reviewed next year
- Define triggers (Independent Price Verification, back testing) that result in immediate model review

Policies, Procedures and Documentation

- Governance of Model Risk Management is the responsibility of the Board of Directors and Senior Management of the Bank
- Policies and Procedures need to cover :
  - model design
  - model validation and periodic review
  - what to do in case of model failure ?
  - compliance (Internal Audit)
- Documentation of the models is paramount
  - It takes time and effort and may seem fastidious for model developers and users who know the model well

⇒ provide incentive to produce quality documentation

- Model Inventory
  - purpose, products, limitations, last calibration,…

Communication

- Model risk is not a "quant" issue: it is a global issue !
- Senior Management and Board of Directors need to be informed
- Communication covers:
  - Performance of Models
    - Back testing (VaR, rating)
    - Benchmarking (pricing and recently internal models)
  - Quantification of uncertainty coming from Model Risk
    - Valuation uncertainty for Mark-to-Model
    - Inadequate capital charge computation
    - Inappropriate valuation of alternative business opportunities
  - Major evolutions of sensitive models
    - Significant changes in model specifications and their implications